

Chapter 3: Patterns, Relations, Equations and Predictions

- Describing Patterns
- Solving Equations (algebra)
- $y = mx + b$

Section 3.1 - Describing Patterns

Curriculum Outcomes	Related Activities	Page in Text
<ul style="list-style-type: none"> • express problems in terms of equations and vice versa • model real-world phenomena with linear, quadratic, exponential, and power equations • gather data, plot the data using appropriate scales, and demonstrate an understanding of independent and dependent variables and domain and range • construct and analyze tables relating two variables • develop and apply strategies for solving problems • describe real-world relationships depicted by graphs and tables of values • identify, generalize, and apply patterns • solve problems using graphing technology • determine if a graph is linear by plotting points in a given situation 	<ul style="list-style-type: none"> • investigations on gathering data about visible faces on cube "trains" • a Focus on graphing data and using data to make predictions • develop an equation in the form $ax + b = c$ • demonstrate and apply an understanding of discrete and continuous number systems 	<p>96</p> <p>97</p> <p>98</p> <p>101</p>

THE NUMBER SYSTEM

W = Whole Numbers

I = Integers

\bar{Q} = Irrational Numbers

R = Real Numbers

N = Natural Numbers

Q = Rational Numbers

EXAMPLES:

W: 0, 1, 2, 3, ...

\bar{Q} : π (3.141592...), $\sqrt{3}$, 1.23456738..., $\sqrt{15}$, ...

N: 1, 2, 3, ...

I: ..., -3, -2, -1, 0, 1, 2, 3, ...

R: $-\frac{1}{2}$, $\sqrt{15}$, 0, -3, 3, π (3.141592), ...

Q: $\frac{1}{2}$, $-\frac{1}{2}$, $\frac{11}{3}$, 0.2, -0.2, 3, -3, 0, ...

• Definitions

- Real numbers (R): ALL numbers; rational & irrational
- Irrational numbers (\bar{Q}):
 - they cannot be written as a fraction
 - non-repeating decimal
 - non-terminating decimal
 - Examples: 0.2163875943.... and π
- Rational numbers (Q):
 - a number that can be written as a fraction
 - Any number that is not an irrational number
 - Examples: -2.34, $3.\overline{456}$, 6.323 232 32...

Definitions continued...

- Integers (I):
 - Positive and negative whole numbers
 - NO decimals
 - Examples: -400, +8, 0, 29, -49578
- Whole numbers (W):
 - all of the positive integers and zero
 - Examples: 0, 1, 2, 3, 4, etc.
 - NO decimals
- Natural numbers (N):
 - all of the positive integers
 - DOES NOT include zero (only difference from whole numbers)
 - Examples: 1, 2, 3, 4, etc.

Using the previous definitions, determine if the following statements are sometimes true, always true, or never true. Justify your choices.

- A) All whole numbers are integers *Always*
- B) All integers are whole numbers *Sometimes*
-1, 0, 1
- C) If a number is an integer then it is also a rational number. *Always*
- D) If a number is a rational number then it is also an integer. *Sometimes*
4 3.2
- E) There is a number which is both rational and irrational. *Never*

Copy and complete the table:

For each of the following numbers in the table, put an "x" in each category that the number belongs to. It may only belong in one, but could also belong to 5 out of the 6 categories. The first one is done for you.

Number	Real	Rational	Irrational	Whole	Natural	Integer
3.2	x	x				
0						
5.66						
-7						
15						
20009						
4.569...						
3.14...						
-3.22						
4/5						
14/2						
-6/3						
5/2						
-4.567...						
-23						
10						

Please double check your answers to make sure that you marked the appropriate boxes.

Number	Real	Rational	Irrational	Whole	Natural	Integer
3.2	X	X				
0	X	X		X		X
5.66	X	X				
-7	X	X				X
15	X	X		X	X	X
20009	X	X		X	X	X
4.569...	X		X			
3.14...	X		X			
-3.22	X	X				
4/5	X	X				
14/2	X	X		X	X	X
-6/3	X	X				X
5/2	X	X				
-4.567...	X		X			
-23	X	X				X
10	X	X		X	X	X

Set Notation:

- We need to know what these signs mean:

such that \rightarrow |
less than \rightarrow <
greater than \rightarrow >
less than or equal to \rightarrow ≤
greater than or equal to \rightarrow ≥
belongs to \rightarrow ∈

- We need to know what number type we are dealing with:

Natural Number = **N**

Whole Numbers = **W**

Integer = **I**

Rational Numbers = **Q**

Irrational Numbers = **\overline{Q}**

Real Numbers = **R**

- Example:

$$\{x / x \leq 5, x \in I\}$$

Set Notation:

- We need to know what these signs mean:

such that \rightarrow |
less than \rightarrow <
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Natural Number = **N**

Whole Numbers = **W**

Integer = **I**

Rational Numbers = **Q**

Irrational Numbers = **\overline{Q}**

Real Numbers = **R**

- Example:

$$\{x / x \leq 5, x \in I\}$$

↑
Variable
(x)
↑
x is
less than
or equal to 5
↑
x belongs to
Integers

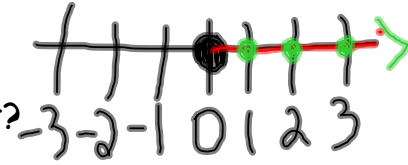
Possible answers: 4, 5, 3, 0, -2

What if we were to graph this on a number line?

~~R~~ W

Ask Yourself:

- What set of numbers am I dealing with?
- What is the sign?: am I going right or left?
- Dots or a line?: Dots Lines



Integers

Real

Natural

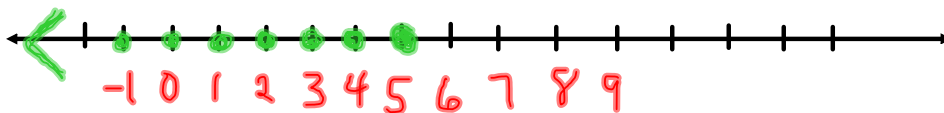
Irrational

Whole

Rational

- Solid or open dots?: Solid Open
 - if it can be equal to - If that number isn't included

$$\{x / x \leq 5, x \in \textcircled{I}\}$$



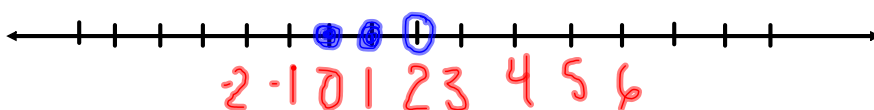
Example #2:

$$\{x / x > 3, x \in \mathbb{R}\}$$



Example #3:

$$\{x / x < 2, x \in \mathbb{W}\}$$



← dots

Class work / Homework:

Copy and Complete the following:

Section 3.1 - "Graphing Number Lines"

1. What set of numbers do the following represent?

- a) \mathbb{N} b) $\overline{\mathbb{Q}}$ c) \mathbb{Q} d) \mathbb{R} e) \mathbb{W} f) \mathbb{I}

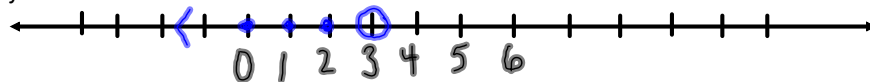
2. Graph the following on a number line. Use a ruler to draw the line.

- | | | |
|---|---|--|
| a) $\{x / x < 3, x \in \mathbb{I}\}$ | b) $\{x / x < 3, x \in \mathbb{R}\}$ | c) $\{x / x \geq 2, x \in \mathbb{N}\}$ |
| d) $\{x / x \geq 2, x \in \mathbb{I}\}$ | e) $\{x / x < -3, x \in \mathbb{R}\}$ | f) $\{x / x < -3, x \in \mathbb{I}\}$ |
| g) $\{x / x < 4, x \in \mathbb{W}\}$ | h) $\{x / x \geq 0, x \in \mathbb{R}\}$ | i) $\{x / 0 < x \in \mathbb{I}\}$ x > 0 |
| j) $\{x / 0 < x, x \in \mathbb{R}\}$ | k) $\{x / 9 > x, x \in \mathbb{R}\}$ | l) $\{x / 9 > x, x \in \mathbb{N}\}$ |

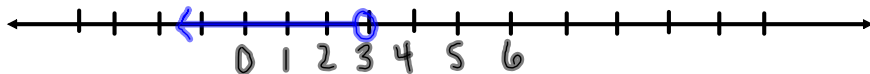
-1 0 . . .

Answers:

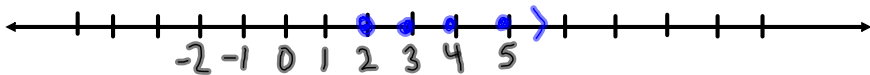
a) $\{x / x < 3, x \in \mathbb{I}\}$



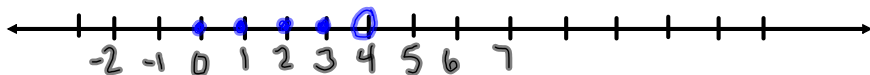
b) $\{x / x < 3, x \in \mathbb{R}\}$



c) $\{x / x \geq 2, x \in \mathbb{N}\}$



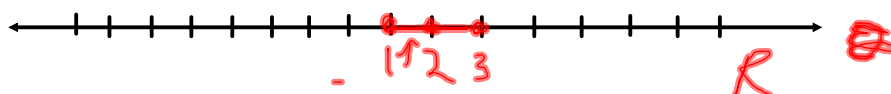
g) $\{x / x < 4, x \in \mathbb{W}\}$



No arrow and
no dots on the negatives
because whole numbers
are not negative.

f,

- a) $\{x / x < 3, x \in \mathbb{I}\}$ b) $\{x / x < 3, x \in \mathbb{R}\}$ c) $\{x / x \geq 2, x \in \mathbb{N}\}$
 d) $\{x / x \geq 2, x \in \mathbb{I}\}$ e) $\{x / x < -3, x \in \mathbb{R}\}$ f) $\{x / x < -3, x \in \mathbb{I}\}$
 g) $\{x / x < 4, x \in \mathbb{W}\}$ h) $\{x / x \geq 0, x \in \mathbb{R}\}$ i) $\{x / 0 < x, x \in \mathbb{I}\}$
 j) $\{x / 0 < x, x \in \mathbb{R}\}$ k) $\{x / 9 > x, x \in \mathbb{R}\}$ l) $\{x / 9 > x, x \in \mathbb{N}\}$



dots
 {
 I
 W
 N
 }
 no decimals

✓

—, ,

State what type of number system each of the following sets of numbers would fall under:

- a) $\{-2, -1, 0, 3, 5, 7\}$ R Q I
 b) $\{-4.5, -2, -0.5, 0, 0.5, 6\}$ R Q
 c) $\{0, 2, 4, 6, 8\}$ R Q I W
 d) $\{2, 4, 6, 8, 10, 12\}$ R Q N W I
 e) $\{1/2, 1/4, 0.75\}$ R Q
 f) $\{\pi, \sqrt{2}, 5.482957271615303846202784\}$

R Q

R

$\bar{\mathbb{Q}}$ or \mathbb{Q}

I, W, N No decimals

10

Warm Up: #10

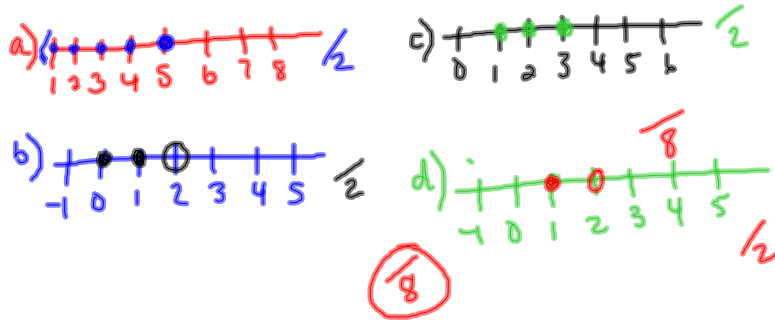
1. State what type of number system each of the following sets of numbers would fall under:

- a) $\{-2, -1, 0, 3, 5, 7\}$
- b) $\{-4.5, -2, -0.5, 0, 0.5, 6\}$
- c) $\{0, 2, 4, 6, 8\}$
- d) $\{2, 4, 6, 8, 10, 12\}$
- e) $\{1/2, 1/4, 0.75\}$
- f) $\{\pi, \sqrt{2}, 5.482957271615303846202784\}$

2. Graph the following on a number line:

a) $\{x / x \leq 5, x \in \mathbb{I}\}$ b) $\{x / x < 2, x \in \mathbb{W}\}$

c) $\{x / x \leq 3, x \in \mathbb{N}\}$ d) $\{x / x < 2, x \in \mathbb{N}\}$



10

Warm Up: #10

1. State what type of number system each of the following sets of numbers would fall under:

- a) $\{-2, -1, 0, 3, 5, 7\}$ $\mathbb{R} \mathbb{Q} \mathbb{I}$
- b) $\{-4.5, -2, -0.5, 0, 0.5, 6\}$ $\mathbb{R} \mathbb{Q}$
- c) $\{0, 2, 4, 6, 8\}$ $\mathbb{R} \mathbb{Q} \mathbb{I} \mathbb{W}$
- d) $\{2, 4, 6, 8, 10, 12\}$ $\mathbb{R} \mathbb{Q} \mathbb{I} \mathbb{W} \mathbb{N}$
- e) $\{1/2, 1/4, 0.75\}$ $\mathbb{R} \mathbb{Q}$
- f) $\{\pi, \sqrt{2}, 5.482957271615303846202784\}$ $\mathbb{R} \mathbb{Q}$

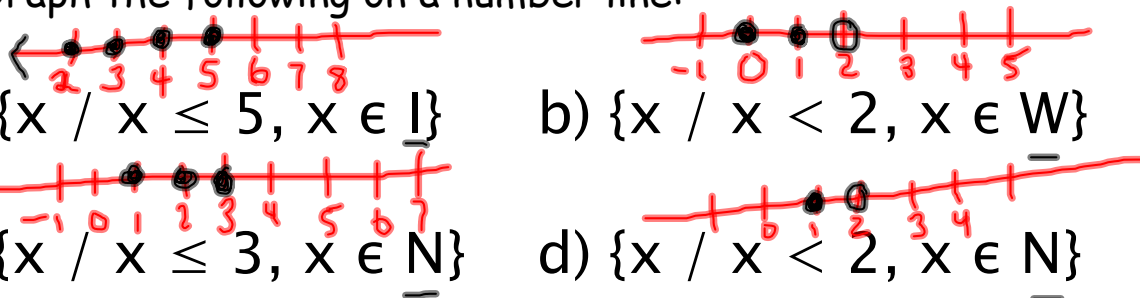
2. Graph the following on a number line:

a) $\{x / x \leq 5, x \in \mathbb{I}\}$

b) $\{x / x < 2, x \in \mathbb{W}\}$

c) $\{x / x \leq 3, x \in \mathbb{N}\}$

d) $\{x / x < 2, x \in \mathbb{N}\}$



Modeling:

- a technique of producing a mathematical description or model that can be used to solve a practical problem
- modeling can be done through the use of:

1. Equations

Example: $y = 2x$

2. Table of values

Example:

x	y
0	0
1	2
2	4
3	6

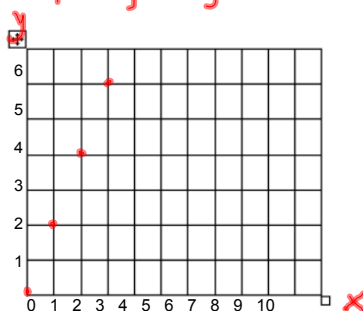
(0,0)
(1,2)

3. Ordered Pairs

Example: (0, 0) (1, 2) (2, 4) (3, 6)

4. Graphing

Example:



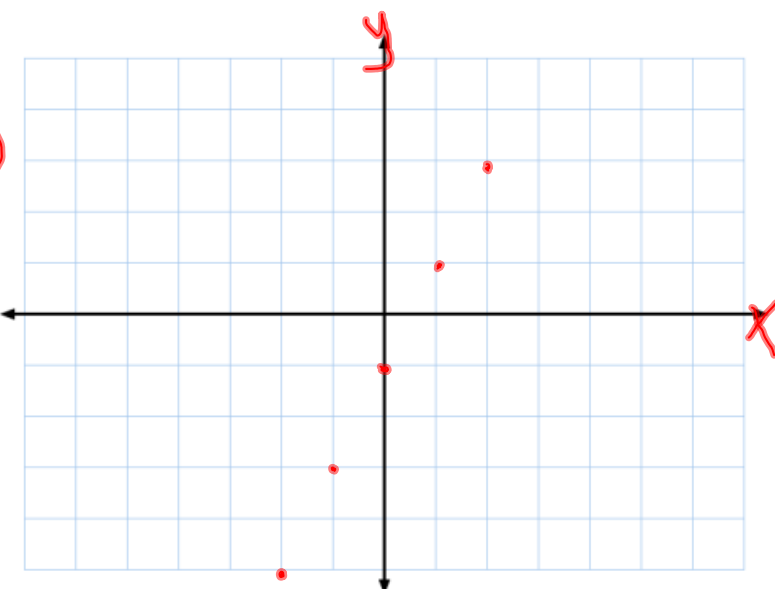
Graphing Review:

Graph the following: $y = 2x - 1$ $x, y \in \mathbb{R}$

1. Table of Values:

x	y
-2	-5
-1	-3
0	-1
1	1
2	3

(-2, -5)
(-1, -3)
(0, -1)
(1, 1)
(2, 3)



2. Graph the co-ordinates:

Domain & Range:

Domain - set of all possible x values

Range - set of all possible y values

- When writing domain and range in set notation they should be written in order of smallest to largest. Numbers should not be repeated.

What is the domain for the following ordered pairs? What is the range?

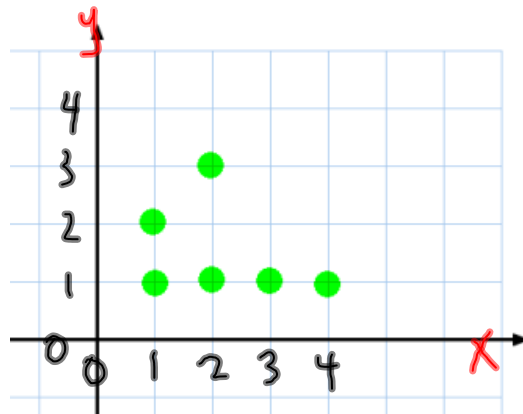
$(2,1) (3,4) (5,6) (8,9) (10,11)$ ← $(3,9)$

Domain = $2, 3, 5, 8, 10$

Range = $1, 4, 6, 9, 11$

What is the domain for the following graph? What is the range?

$(x,y) \rightarrow$ always this order.



- least to greatest
- can't repeat #'s

Domain^x = $1, 2, 3, 4$

Range^y = $1, 2, 3$

Discrete & Continuous:

Discrete Data

- finite number of values in between 2 points
- every number is not possible
- easily "countable"
- dots on a graph

Examples:

- o Number of books on a shelf
- o Number of defective items in a shipment of 50 pens

W, N, I
- - - - -
 ↑

Continuous Data

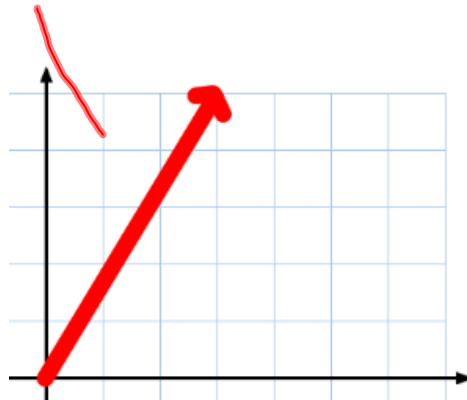
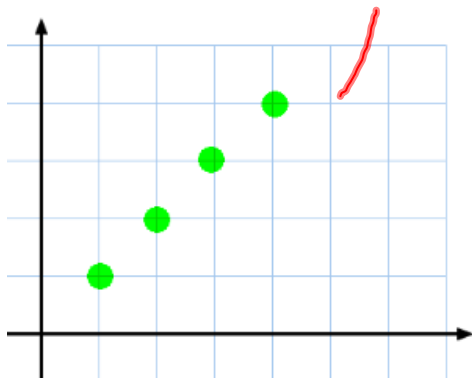
- infinite number of values in between 2 points
- every number is possible
- dots are joined

Examples:

- o 1-5 and everything in between
- timing for a 100 m dash

R

Is this a graph of Discrete or Continuous Data?



Practice:

1. What is the domain & range of the following set of ordered pairs?

(2,1) (3,2) (8,9) (3,10) (1,3) (3,6) (2,10)

2. Are the following situations discrete or continuous?

- a) The height of trees at a nursery over a period of 20 years C
- b) The number of correct answers on a student's multiple choice quiz D
- c) How many times it would take a person to pass their driver's test D
- d) The length of time it takes for a light bulb to burn out C

$$\cancel{2x} - y = 2$$

$\cancel{-2x} \quad -2x$

Example

$$y = \underline{mx} + \underline{b}$$

$$x, y \in \underline{W}$$

$$\begin{array}{r} -1y = -2x + 2 \\ \hline -1 \quad -1 \end{array}$$

$$y = \underline{2x - 2}$$

x	y
0	-2
1	0
2	2
3	4
4	6

~~(0, -2)~~ (1, 0) (2, 2)
 (3, 4) (4, 6)

$$2x - 2$$

$$2(0) - 2$$

$$0 - 2$$

$$-2$$

$$2(1) - 2$$

$$2 - 2$$

$$= 0$$

Class work / Homework:

Complete worksheet:

"Section 3.1 - Domain & Range, Discrete & Continuous"

Domain, Range, Continuous & Discrete worksheet #2.doc

Questions:

1. List the domain and range in set notation for each of the following sets of ordered pairs.

a) $\{(2,1), (-1, 3), (4, 2), (3,-2)\}$

$$d = \{x \mid x = -1, 2, 3, 4, x \in \mathbb{I}\}$$

$$r = \{y \mid y = -2, 1, 2, 3, y \in \mathbb{I}\}$$

b) $\{(0,2), (-1,-1), (3,2), (2,3)\}$

$$d = \{x \mid x = -1, 0, 2, 3, x \in \mathbb{I}\}$$

$$r = \{y \mid y = -1, 2, 3, y \in \mathbb{I}\}$$

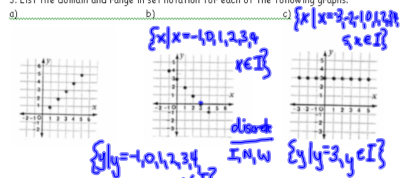
2. List the domain and range in set notation for the table of values.

X	3	3	4	5	3	4	5	4	5	5
Y	1	1	1	1	2	2	2	3	3	4

$$\{x \mid x = 3, 4, 5, x \in \mathbb{N}\}$$

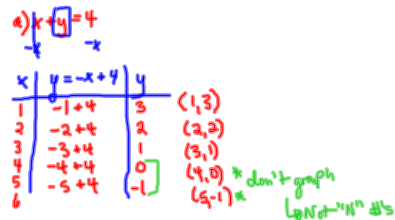
$$\{y \mid y = 1, 2, 3, 4, y \in \mathbb{N}\}$$

3. List the domain and range in set notation for each of the following graphs.

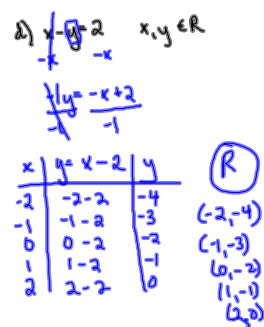


4. Graph the following equations by creating a table of values. Determine if the data is either discrete or continuous.

- a) $x + y = 4$ $x, y \in \mathbb{R}$ b) $x + y = 3$ $x, y \in \mathbb{I}$
 c) $x + y = 2$ $x, y \in \mathbb{R}$ d) $x - y = 2$ $x, y \in \mathbb{R}$



discrete or continuous
 Why? \mathbb{N} - No decimals

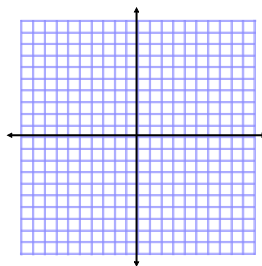


Example 1: Make a table of values and graph

$y = 4x - 5$ $x, y \in \mathbb{I}$ \rightarrow "x" values must be:

x	y = 4x - 5	y
-2	$y = 4(-2) - 5$	

Coordinates:



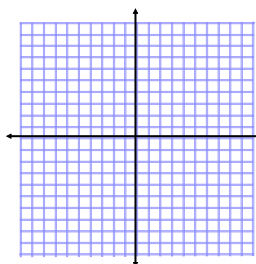
Example 2: $x - y = 5$ $x, y \in \mathbb{W}$

- You need to rearrange this so that it is $y =$ ____

$x - y = 5$

x	y
---	---

Coordinates:



Class work / Homework:

For each of the following make a table of values and graph the coordinates. Copy these down.

continuous
or
discrete

(1) $y = -2x - 1$ $x, y \in \mathbb{I}$

(2) $y = 3x + 2$ $x, y \in \mathbb{W}$

(3) $y = -1x - 2$ $x, y \in \mathbb{N}$

(4) $x + y = -2$ $x, y \in \mathbb{R}$

(5) $-5y + 5x = 10$ $x, y \in \mathbb{I}$

Attachments

Domain, Range, Continuous & Discrete worksheet #2.doc